

Similarity-Based Logics for Approximate Entailments

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Abstract Reasoning under practical circumstances is often inexact. Assumptions might be fulfilled only in an approximate way but conclusions are drawn anyway. Different epistemic aspects may be involved, like uncertainty, preference or similarity. In order to formalise such kind of reasoning we need to go beyond classical propositional logic. In this presentation we will deal with logics for similarity-based reasoning. This kind of reasoning can be cast in the more general framework of reasoning by analogy and has applications, for example, in classification, case-based reasoning, or interpolation.

In his seminal work on similarity-based reasoning, Ruspini proposes [5] the interpretation of fuzzy sets in terms of (crisp) sets and fuzzy similarity relations. To this end, he builds up a framework for approximate inference that is based on the mutual similarity of the propositions involved. Following these lines, a number of approaches have dealt with fuzzy similarity-based reasoning from a logical perspective. In particular, the so-called logic of *Approximate Entailment* (LAE), and the logic of a dual notion of *Strong Entailment* (LSE) have been studied [2, 7]. These logics formalise the effect of small changes on the validity of logical relationships. For instance, consider a pair of propositions such that none is implied by the other one; we may then still ask if one proposition is a consequence of the other one by means of a slight change. Conversely, for a pair of propositions one of which is a consequence of the other one, we may ask if this consequence relation is stable under small changes.

To formalise these kind of inferences, one needs to specify what is meant by “*approximate*”. Several approaches dealing with statements interpreted in metric spaces or logics on comparative similarity have also been considered in the literature, see e.g. [1, 4, 6]. Here we will follow a quantitative approach and use fuzzy similarity relations to model the notion of approximate entailment [3]. In this talk we will present the main notions and properties of LAE, a propositional graded modal logic where propositions are interpreted, as in classical logic, by subsets of a fixed set,

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called the set of worlds, that in addition it is assumed to be endowed with a fuzzy similarity relation, which associates with each pair of two worlds their degree of resemblance. The basic semantic structures are hence *fuzzy similarity spaces*, which consist of a set of worlds and a fuzzy similarity relation, and the core syntactic objects of LAE are implications between propositions endowed with a degree. The intended meaning of a statement of the form $A >_c B$ is that B is an approximate consequence of A to the degree c , where c is a real number between 0 and 1. We will present a complete axiomatization, and moreover we will show some extensions as well as how the framework can be enhanced in case of dealing with similarities on linearly ordered scales [8].

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