

- PETR CINTULA, ZUZANA HANIKOVÁ, ROSTISLAV HORČÍK, CARLES NOGUERA, *Non-associative substructural logics: alternative axiomatization, algebraic and logical properties.*

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Galatos and Ono have studied in a recent paper (Cut elimination and strong separation for substructural logics: An algebraic approach, *Annals of Pure and Applied Logic*, 161(9):1097–1133, 2010) a non-associative version of full Lambek calculus, which we call here SL. They have introduced a Gentzen-style and a Hilbert-style calculus for that logic and prove that it is algebraizable and its equivalent algebraic semantics is the variety of lattice-ordered residuated unital groupoids. On the other hand, Cintula and Noguera have developed (in the second chapter of the *Handbook of Mathematical Fuzzy Logic - volume 1*, Studies in Logic, Mathematical Logic and Foundations, vol. 37, College Publications, London, 2011) a general algebraic framework to deal with substructural logics with SL as the base logic. They introduce the notion of almost (MP)-based logic (a logic with a Hilbert-style presentation where *modus ponens* is the only binary rule, there are no rules with more than two premises, and all unary rules are of the form $\varphi \vdash \delta(\varphi)$, for $\delta \in \text{DT}$, where the set of terms DT satisfies some good properties; the logic is (MP)-based if $\text{DT} = \emptyset$) and prove that every almost (MP)-based substructural logic enjoys a local deduction theorem and a certain form of proof by cases property (PCP). The main associative substructural logics (FL, FL_e, FL_{ew}, etc.) are (almost) (MP)-based. However the problem was left open for SL and other non-associative logics. In this talk we positively solve this problem. We present an alternative Hilbert-style axiomatization of SL which allows to show that it is almost (MP)-based, and hence the same holds for all its axiomatic extensions. We obtain several interesting logical and algebraic consequences of this fact: a form of the local deduction theorem, a description of intersection of filters and of the filter generated by a given set, an axiomatization of intersection of two axiomatic extensions of a given logic, and equational bases of varieties of SL-algebras generated by positive universal classes of SL-algebras. Finally, as another by-product of our alternative Hilbert system for SL, we show how to axiomatize the logic of linearly ordered residuated unital groupoids.

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