

- PETR CINTULA, CARLES NOGUERA, *Generalized Disjunctions and (Infinitary) Structural Consequence Relations*.

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This paper is an abstract investigation on disjunction connectives in an Abstract Algebraic Logic approach. The most comprehensive treatment so far, summarizing and generalizing previous works, was presented by Czelakowski in his monograph “Protoalgebraic Logics” (Kluwer, 2000) where he considered generalized disjunction connectives given by a (possibly parameterized and infinite) set of formulae in two variables satisfying the Proof by Cases Property (PCP). This approach allowed to obtain several interesting characterizations in terms of other properties of logics and their semantics, namely: distributivity of the lattice of theories, distributivity of the lattice of filters over any algebraic model of the logic and the behavior of prime filters and substitutions. However, all these results were restricted to *finitary* logics. This paper develops the theory mainly by generalizing it to *infinitary* logics. In particular:

- We identify a strong form of the proof by cases property (sPCP), which is equivalent to the PCP for finitary logics.
- We present a hierarchy of logics based on (1) the form of the disjunction they possess (given by a single formula, a set of formulae, or a parameterized set of formulae) and on (2) the kind of proof by cases property this disjunction satisfies (wPCP, PCP, or sPCP).
- We generalize the known results in two ways: first we prove their *modified* versions (e.g. we replace PCP by sPCP and the distributivity of the lattice of filters with the condition of it being a frame) which hold in complete generality and coincide with known ones in the finitary case. Second, we extend the validity of known results from finitary logics to the strictly larger class of logics with the IPEP (intersection-prime extension property), i.e. logics where the finite meet-irreducible theories form a basis of the closure system of theories. This class includes not only all finitary logics, but also all semilinear logics (e.g. infinitary Łukasiewicz logic).
- We show several consequences of the presence of a suitable disjunction in a given logic. Namely, we can find an axiomatization of the extension of this logic semantically defined by a positive universal class of its models, and as a particular case we show how to axiomatize the intersection of any finite set of its axiomatic extensions.

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