Mechanism Design for Double Auctions with Temporal Constraints

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Contributions

- Double Auction with Temporal Constraints
  1. Multiple buyers and sellers,
  2. Type: valuation + time constraint \([t,t']\)

- Bipartite Graph

- VCG Mechanism

- Efficient Allocation

- Min-Max Payment

- Lemma 3

- Clark Pivot Payment

- Theorem 2

- Theorem 3

- Theorem 4

- Efficient Allocation

- Monotonic

- Critical Value

- Efficient Bipartite Matching Allocation

- Max-weighted Bipartite Matching Allocation

- Min-Max Payment

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- Clark Pivot Payment
Outline

1. The Model
   - The Domain
   - The Question
   - A Solution

2. Augmentation-based Mechanism
   - Graphical Representation
   - Allocation Policy Design
   - Payment Policy Design

3. Conclusion
Outline

1. The Model
   - The Domain
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   - A Solution

2. Augmentation-based Mechanism

3. Conclusion
The Domain

Double Auction with Temporal Constraints

1. Multiple buyers and sellers,
2. Type(private): valuation + time constraint \([t, t']\)
Efficient & Truthful Mechanism

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint \([t,t']\)
Components of the Mechanism

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint [t,t']

(v1,t1,t2)  (v3,t5,t6)
(v2,t3,t4)
...

Mechanism

Efficient  Truthful

Allocation Policy

Payment Policy
Achieving Efficiency

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint \([t, t']\)

\((v_1, t_1, t_2)\) \((v_2, t_3, t_4)\) \((v_3, t_5, t_6)\) ...

Mechanism
Efficient
Truthful

Allocation Policy
Efficient

Payment Policy
A Solution

Achieving Truthfulness

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint \([t,t']\)

\[ (v_1,t_1,t_2) \quad (v_3,t_5,t_6) \]
\[ (v_2,t_3,t_4) \quad ... \]

Mechanism

- Efficient
- Truthful

Allocation Policy

- Efficient
- Monotonic

Payment Policy

- Critical Value

[Refs: [Nisan, 2007], [Parkes, 2007]]
The VCG Mechanism

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint $[t, t']$

(v1, t1, t2)    (v3, t5, t6)
(v2, t3, t4)
...

Mechanism
Efficient
Truthful

Allocation Policy
Efficient
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Payment Policy
Critical Value

VCG Mechanism
Efficient Allocation
Clark Pivot Payment
Outline

1. The Model
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   - Graphical Representation
   - Allocation Policy Design
   - Payment Policy Design
3. Conclusion
Constructing the Graph

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
2. Type: valuation + time constraint \([t,t']\)

Bipartite Graph

Mechanism
- Efficient
- Truthful

Allocation Policy
- Efficient
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Payment Policy
- Critical Value

VCG Mechanism
- Efficient Allocation
- Clark Pivot Payment
Constructing the Graph (an Example)

An ask $\theta_i = (v_i, s_i, e_i)$ and a bid $\theta_j = (v_j, s_j, e_j)$ are \textbf{matchable} (i.e. item exchanging can happen between $i$ and $j$) iff $v_i \leq v_j$ and $[s_i, e_i] \cap [s_j, e_j] \neq \emptyset$.

Asks
- (2,3,6)
- (3,7,8)
- (4,5,12)
- (5,11,12)

Bids
- (10,5,10)
- (9,2,4)
- (8,1,3)
- (7,9,11)
Definition

Given a graph $G$, a **matching** $M$ in $G$ is a set of pairwise non-adjacent edges, i.e. no two edges share a common vertex.
Matching & Alternating Paths

Given a matching $M$,

- an $M$-alternating path is a path in which the edges belong alternatively to $M$ and not to $M$. 
Given a matching $M$,

- an $M$-augmenting path is an $M$-alternating path whose endpoints are free/unmatched. $(3, 10, 2, 9)$
Given a matching $M$,

- an **$M$-abridging path** is an $M$-alternating path whose first edge and last edge are in $M$. (2,10,4,7)
Given a matching $M$, 

- an $M$-replacement path is an $M$-alternating path where one of the endpoints is free and one of the ending edges is in $M$. $(2, 10, 4, 7, 5)$
Maximum-weighted Bipartite Matching Allocation

Double Auction with Temporal Constraints
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Bipartite Graph

(v1,t1,t2) (v3,t5,t6)
(v2,t3,t4)
...

Mechanism
- Efficient
- Truthful

Allocation Policy
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Max-weighted Bipartite Matching Allocation

VCG Mechanism
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Clark Pivot Payment
Maximum-weighted Bipartite Matching
The Allocation Policy

**Initialization:**
- Encode reports $\theta$ in bipartite graph $G_\theta$.
- Set the result matching $M = \emptyset$ for $G_\theta$.

**Recursion:**
- $\text{AugPath} = \{p : \Delta(p) > 0 \text{ and } p \text{ is an } M\text{-augmenting path}\}$.
- $\text{MaxAugPath} = \arg \max_{p \in \text{AugPath}} \Delta(p)$.
- If $\text{MaxAugPath} = \emptyset$, stop recursion.
- Otherwise, let $\hat{p} \in \text{MaxAugPath}$ s.t. $p \preceq_p \hat{p}$ for any $p \in \text{MaxAugPath}$, and $M = M \oplus \hat{p}$.
Properties of The Allocation

Double Auction with Temporal Constraints
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Bipartite Graph

Mechanism
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Max-weighted Bipartite Matching Allocation

Theorem 2

Theorem 3

VCG Mechanism

Efficient Allocation

Clark Pivot Payment
### Min-Max Payment

Double Auction with Temporal Constraints
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#### Theorem 2

#### Theorem 3

### Bipartite Graph

#### VCG Mechanism

#### Efficient Allocation

#### Clark Pivot Payment

#### Mechanism
- ***Efficient***
- ***Truthful***

#### Allocation Policy
- ***Efficient***
- ***Monotonic***

#### Payment Policy
- ***Critical Value***

### Max-weighted Bipartite Matching Allocation

\((v_1,t_1,t_2)\) \((v_2,t_3,t_4)\) \((v_3,t_5,t_6)\) ...
The Model
Augmentation-based Mechanism
Conclusion
Payment Policy Design

The Intuition

For each matched ask/bid, looking for the best substitution and the lowest loss if the ask/bid was not participated.
For an matched ask

- each **abridging path**, *starting from the ask*, gives a way to remove the ask by unmatching another bid,

- each **replacement path**, *starting from the ask*, gives a way to remove the ask by giving a substitution.

The lowest loss

The best substitution
The Payment Policy

\[ x_i(\theta) = \begin{cases} 
\min_{p \in A \cup R} v(\text{ending}(p)), & \text{if } i \text{ is seller} \\
\max_{p \in A \cup R} v(\text{ending}(p)), & \text{if } i \text{ is buyer} 
\end{cases} \]

where

- \( A \) is a set of all **abridging** paths starting from \( \theta_i \),
- \( R \) is a set of all **replacement** paths starting from \( \theta_i \),
- and \( v(\text{ending}(p)) \) is the valuation of the ending vertex, the endpoint other than \( \theta_i \), of path \( p \).
For an matched ask

the lowest loss

the best substitution
Properties of the Payment Policy

- **Efficient** Allocation Policy
  - Efficient
  - Monotonic
- **Payment Policy**
  - Critical Value
- **Mechanism**
  - Efficient
  - Truthful
- **Theorem 2**
- **Theorem 3**
- **Theorem 4**
- **Max-weighted Bipartite Matching Allocation**
- **Min-Max Payment**
- **VCG Mechanism**
- **Efficient Allocation**
- **Clark Pivot Payment**
- **Lemma 3**
does NOT need to rerun the allocation for each matched ask/bid,

- can be implemented $O(n)$ times faster than Clarke pivot payment.
Outline

1. The Model
2. Augmentation-based Mechanism
3. Conclusion
An efficient and truthful (VCG) double auction with time constraints:

- graphical representation of constraints
- augmentation-based policies
- a faster algorithm for computing VCG payment (critical value)
Thank you for your attention!

Double Auction with Temporal Constraints
1. Multiple buyers and sellers,
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Theorem 2

VCG Mechanism

Efficient Allocation

Theorem 4

Min-Max Payment

Lemma 3

Clark Pivot Payment

Bipartite Graph

Efficient Allocation

Critical Value

Monotonic

Efficient

Min-Max Payment

Efficient Allocation

Critical Value

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